

Asynchronous Reactive Programming with Modal Types in Haskell

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Abstract. The implementation of asynchronous systems, in particular graphical user interfaces, is traditionally based on an imperative model that uses shared mutable state and callbacks. While efficient, the combination of shared mutable state and callbacks is notoriously difficult to reason about and prone to errors. Functional reactive programming (FRP) provides an elegant alternative and recent theoretical advances in modal FRP suggest that it can be efficient as well.

In this paper, we present `Async Rattus`, an FRP language embedded in Haskell. The distinguishing feature of `Async Rattus` is a modal type constructor that enables the composition of asynchronous subsystems by keeping track of each subsystem’s clock at compile time which in turn enables dynamically changing clocks at runtime. The central component of our implementation is a Haskell compiler plugin that, among other aspects, checks the stricter typing rules of `Async Rattus` and infers compile-time clocks. This is the first implementation of an asynchronous modal FRP language. By embedding the language in Haskell we can exploit the existing language and library ecosystem as well as rapidly experiment with new language features and library design. We hope that such experimentation with `Async Rattus` sparks further research in modal FRP and its applications.

1 Introduction

Functional reactive programming (FRP) [14] provides an elegant, high-level programming paradigm for reactive systems. This is achieved by making time-varying values (also called *signals* or *behaviours*) first-class objects that are easily composable. For example, assuming a type `Sig a` that describes signals of type `a`, an FRP library may provide a function `map :: (a → b) → Sig a → Sig b` that allows us to manipulate a given signal by applying a function to it.

Haskell has a rich ecosystem of expressive and flexible FRP libraries [1, 7, 8, 15, 19, 22, 30–32, 35]. Devising such FRP libraries is challenging as its API must be carefully designed to ensure that reactive programs are *causal* and are not prone to *space leaks*. A reactive program is causal if the value of any output signal at any time `t` only depends on the value of input signals at time `t` or earlier. Due to the high-level nature of FRP programs, they can suffer from *space leaks*, i.e. they keep data in memory for too long. Haskell FRP libraries tackle these issues by providing a set of *abstract* types (i.e. their definitions are not exposed)

to represent signals, signal functions, events etc. and only expose a carefully selected set of combinators to manipulate elements of these types.

Over the last decade an alternative to this library-based approach has been developed [3, 4, 21, 24, 26, 28, 29] that uses a modal type operator \bigcirc (pronounced “later”) to express the passage of time at the type level. This type modality allows us to distinguish a value of type a , which is available now, from a value of type $\bigcirc a$, which represents data of type a arriving in the next time step. A language with such a modal type operator \bigcirc has been recently implemented as an embedded language in Haskell called **Rattus** [2].

In **Rattus**, signals can be implemented as follows:¹

```
data Sig a = a ::: ( $\bigcirc$ (Sig a))
```

That is, a signal of type *Sig a* consists of a value of type a now and a signal of type *Sig a* later, thus separating consecutive values of the signal by one time step. Instead of hiding the definition of *Sig* from the user, **Rattus** ensures the operational guarantees of causality and absence of space leaks via its type system.

However, the use of the \bigcirc modality limits **Rattus** to *synchronous* reactive programs where all components of the program progress according to a *global clock*. This is witnessed by the fact that we can implement the following function that takes two delayed integers and produces their delayed sum:

```
add ::  $\bigcirc$ Int  $\rightarrow$   $\bigcirc$ Int  $\rightarrow$   $\bigcirc$ Int
add x y = delay (adv x + adv y)
```

This only works because the two delayed integers x and y are guaranteed to arrive at the same time, namely the next tick of the global clock.

Computing according to a *global clock* is a reasonable assumption for many contexts such as simulations and games as well as typical application domains of synchronous (dataflow) languages [6, 9, 33] such as real-time and embedded systems. However, for many applications, e.g. GUIs and concurrent systems, the notion of a global clock may not be natural and may lead to inefficiencies.

In this paper, we present **Async Rattus**, an embedded modal FRP language that replaces the *single global clock* of **Rattus** with *dynamic local clocks* that enable asynchronous computations. **Async Rattus** is based on the **Async RaTT** calculus for asynchronous FRP that has recently been proposed by Bahr and Møgelberg [5] and has been shown to ensure causality and absence of space leaks. Moreover, **Async Rattus** is implemented as a shallowly embedded language in Haskell, which means that **Async Rattus** programs can seamlessly interact with regular Haskell code and thus also have access to Haskell’s rich library ecosystem.

Similarly to **Rattus**, the implementation of **Async Rattus** consists of a library that implements the primitives of the language along with a plugin for the Glasgow Haskell Compiler (GHC). The plugin is needed to check the language’s more restrictive variable scope rules and to ensure the eager evaluation strategy that is necessary to obtain the operational properties. However, **Async Rattus** requires

¹ The data constructor `:::` for *Sig* is written as an infix operator, similarly to `:` for lists.

an additional novelty: The underlying core calculus of `Async Rattus` requires explicit *clocks annotations* in the program. These annotations are necessary to keep track of the dynamic data dependencies in FRP programs. Our implementation of `Async Rattus` infers these clock annotations and transforms the intermediate code generated by GHC from the `Async Rattus` code accordingly.

The remainder of the paper is structured as follows: Section 2 describes the syntax and semantics of `Async Rattus` with a particular focus on its non-standard typing rules. Section 3 illustrates the expressiveness of `Async Rattus` and its interaction with Haskell with the help of a selection of example programs. Section 4 describes how `Async Rattus` is implemented as an embedded language in Haskell. Finally, Section 5 and section 6 discuss related and future work, respectively.

`Async Rattus` is available as a package on Hackage [18]. Apart from the language implementation itself, the package also contains an FRP library implemented in `Async Rattus` along with example programs using this library. In particular, it contains the full source code of all examples presented in this paper.

2 Introduction to `Async Rattus`

`Async Rattus` differs from Haskell in two major ways. Firstly, `Async Rattus` is eagerly evaluated. This difference in the operational semantics is crucial for the language’s ability to avoid space leaks. Secondly, `Async Rattus` extends Haskell’s type system with two type modalities, \bigcirc and \square . A value of type $\bigcirc a$ is a delayed computation that waits for an event upon which it will produce a value of type a , whereas a value of type $\square a$ is a thunk that can be forced at any time, now or in the future, to produce a value of type a .

Each value $x :: \bigcirc a$ waits for an event to occur before it can be evaluated to a value of type a . Intuitively, an element of type $\bigcirc a$ is a pair (θ, f) consisting of a (local) *clock* θ and a thunk f , so that f can be forced to compute a value of type a as soon as the clock θ ticks. This intuition is witnessed by the two functions $cl :: \bigcirc a \rightarrow Clock$ and $adv :: \bigcirc a \rightarrow a$ that project out these two components. Conversely, we can construct a value of type $\bigcirc a$ by providing these two components using the function $delay :: Clock \rightarrow a \rightarrow \bigcirc a$. Using these functions, we can implement a function that takes a delayed integer and increments it:

$$\begin{aligned} incr &:: \bigcirc Int \rightarrow \bigcirc Int \\ incr\ x &= delay_{cl(x)}\ (adv\ x + 1) \end{aligned}$$

This makes explicit the fact that the integers produced by the delayed computations $x :: \bigcirc Int$ and $incr\ x :: \bigcirc Int$ become available at the same time. We write the first argument of `delay` as a subscript. As we will see shortly, these clock arguments are annotations that can always be inferred from the context.

2.1 Typing rules for delayed computations

The type signatures that we have given for `delay` and `adv` above are a good starting point to understand what `delay` and `adv` do, but they are too permissive

$$\begin{array}{c}
\frac{\Gamma, \check{\nu}_{\text{cl}(t)} \vdash t :: A}{\Gamma \vdash \text{delay}_{\text{cl}(t)} t :: \bigcirc A} \quad \frac{\check{\nu} \notin \Gamma' \text{ or } A \text{ stable}}{\Gamma, x :: A, \Gamma' \vdash x :: A} \quad \frac{\Gamma \vdash t :: \bigcirc A}{\Gamma \vdash \text{unbox } t :: A} \quad \frac{\Gamma^\square \vdash t :: A}{\Gamma \vdash \text{box } t :: \bigcirc A} \\
\\
\frac{\Gamma \vdash s :: \bigcirc A \quad \Gamma \vdash t :: \bigcirc B \quad \check{\nu} \notin \Gamma'}{\Gamma, \check{\nu}_{\text{cl}(s) \sqcup \text{cl}(t)}, \Gamma' \vdash \text{select } s t :: \text{Select } A B} \quad \frac{\Gamma \vdash t :: \bigcirc A \quad \check{\nu} \notin \Gamma'}{\Gamma, \check{\nu}_{\text{cl}(t)}, \Gamma' \vdash \text{adv } t :: A} \quad \frac{}{\Gamma \vdash \text{never} :: \bigcirc A} \\
\\
\text{where} \quad \begin{array}{l} \cdot^\square = \cdot \\ (\Gamma, \check{\nu}_\theta)^\square = \Gamma^\square \end{array} \quad (\Gamma, x :: A)^\square = \begin{cases} \Gamma^\square, x :: A & \text{if } A \text{ stable} \\ \Gamma^\square & \text{otherwise} \end{cases}
\end{array}$$

Fig. 1. Select typing rules for Async Rattus.

and we have to reign them in to ensure that Async Rattus programs are causal and do not cause space leaks. If `delay` was simply a function of type $\text{delay} :: \text{Clock} \rightarrow a \rightarrow \bigcirc a$, we could delay arbitrary computations – and the data they depend on – into the future, which will cause space leaks. Figure 1 shows the most important typing rules of Async Rattus.

Async Rattus uses a *Fitch-style* type system [11], which manifests itself by the presence of *tokens* of the form $\check{\nu}_\theta$ (pronounced “tick of clock θ ” or just “tick”) in typing contexts. We can think of $\check{\nu}_\theta$ as denoting the passage of one time step on the clock θ , i.e. all variables to the left of $\check{\nu}_\theta$ are one time step older w.r.t. the clock θ compared to those to the right of $\check{\nu}_\theta$. The rule for `delay` introduces a token $\check{\nu}_{\text{cl}(t)}$ in the typing context Γ . This means that t sees the variables in Γ as one time step older w.r.t. clock $\text{cl}(t)$, thus matching the intuitive semantics of `delay` which delays evaluation of t by one time step on the clock $\text{cl}(t)$.

The variable introduction rule explains how ticks influence which variables are in scope: A variable occurring to the left of a tick is no longer in scope unless it is of a type that is time-independent. We call these time-independent types *stable* types, and in particular all base types such as *Int* and *Bool* are stable as are any types of the form $\bigcirc a$. For instance, function types are not stable, and thus functions cannot be moved into the future, which means that the type checker must reject the following definition:

$$\begin{array}{l}
\text{mapLater} :: (a \rightarrow b) \rightarrow \bigcirc a \rightarrow \bigcirc b \\
\text{mapLater } f \ x = \text{delay}_{\text{cl}(x)} (f (\text{adv } x)) \quad \text{-- } f \text{ is out of scope}
\end{array}$$

The problem is that functions may store time-dependent data in their closure and thus moving functions into the future could lead to space leaks. We shall return to stable types later when we discuss the \square type modality in section 2.2.

Also `adv` cannot be simply a function of type $\bigcirc a \rightarrow a$ as this would allow us to simply execute future computations now, which would break causality. Instead, the typing rule for `adv` only allows us to advance a delayed computation $t :: \bigcirc A$, if we know that the clock of t has already ticked, which is witnessed by the token $\check{\nu}_{\text{cl}(t)}$ in the context. That is, `delay` looks ahead one time step on a clock θ and `adv` then allows us to go back to the present. Variable bindings

made in the future, i.e. the context Γ' in the typing rule for `adv`, are therefore not accessible once we returned to the present.

We can now see why the `add` function from section 1 does not type check:

$$\begin{aligned} \text{add} &:: \bigcirc \text{Int} \rightarrow \bigcirc \text{Int} \rightarrow \bigcirc \text{Int} \\ \text{add } x \ y &= \text{delay}_{\theta}(\text{adv } x + \text{adv } y) \quad \text{-- no suitable clock } \theta \end{aligned}$$

The problem is that there is no clock θ so that both subexpressions `adv x` and `adv y` type check. The former only type checks if $\theta = \text{cl}(x)$ and the latter only type checks if $\theta = \text{cl}(y)$. It might very well be that the clocks of x and y are the same at runtime, e.g. if $y = \text{incr } x$, but that is not guaranteed at compile time.

In order to deal with more than one delayed computation, `Async Rattus` provides the `select` primitive, which takes two delayed computations $s :: \bigcirc A$ and $t :: \bigcirc B$ as arguments, given that a tick on the clock $\text{cl}(s) \sqcup \text{cl}(t)$ is in the context. It produces a value of type `Select A B`, which is defined as follows:²

$$\mathbf{data} \text{ Select } a \ b = \text{Fst } a \ (\bigcirc b) \mid \text{Snd } (\bigcirc a) \ b \mid \text{Both } a \ b$$

A clock of the form $\theta \sqcup \theta'$ ticks whenever θ or θ' ticks. That is, `select s t` waits for a tick on either of the two clocks $\text{cl}(s)$ and $\text{cl}(t)$ and depending on whether $\text{cl}(s)$ ticks before, after, or at the same time as $\text{cl}(t)$, it returns `Fst`, `Snd`, or `Both`, respectively. For example, the following function waits for two integers and returns the integer that arrives first:

$$\begin{aligned} \text{first} &:: \bigcirc \text{Int} \rightarrow \bigcirc \text{Int} \rightarrow \bigcirc \text{Int} \\ \text{first } x \ y &= \text{delay}_{\text{cl}(x) \sqcup \text{cl}(y)} (\mathbf{case} \ \text{select } x \ y \ \mathbf{of} \ \text{Fst } \ x' \ _ \rightarrow x' \\ &\quad \text{Snd } \ _ \ y' \rightarrow y' \\ &\quad \text{Both } x' \ _ \rightarrow x') \end{aligned}$$

With the help of `select`, we can also implement the `add` function from the introduction, but we have to revise the return type:

$$\begin{aligned} \text{add} &:: \bigcirc \text{Int} \rightarrow \bigcirc \text{Int} \rightarrow \bigcirc (\text{Int} \oplus \bigcirc \text{Int}) \\ \text{add } x \ y &= \text{delay}_{\text{cl}(x) \sqcup \text{cl}(y)} (\mathbf{case} \ \text{select } x \ y \ \mathbf{of} \\ &\quad \text{Fst } \ x' \ y' \rightarrow \text{Inr } (\text{delay}_{\text{cl}(y')} (x' + \text{adv } y')) \\ &\quad \text{Snd } \ x' \ y' \rightarrow \text{Inr } (\text{delay}_{\text{cl}(x')} (\text{adv } x' + y')) \\ &\quad \text{Both } x' \ y' \rightarrow \text{Inl } (x' + y')) \end{aligned}$$

where \oplus is the (strict) sum type with the two constructors $\text{Inl} :: a \rightarrow a \oplus b$ and $\text{Inr} :: b \rightarrow a \oplus b$. The type now reflects the fact that we might have to wait two ticks (of two different clocks) to obtain the result. From now on we will elide the clock annotations for `delay` as it will always be obvious from the context what the annotation needs to be. Indeed, `Async Rattus` will infer the correct clock annotation and insert it automatically during compilation.

² `Async Rattus` is a strict language and all type definitions are strict by default.

2.2 Typing rules for stable computations

As we have seen above, only variables of *stable types* can be moved across ticks and thus into the future. A type A is stable if all occurrences of \bigcirc and function types in A are guarded by \square . For example $Int \oplus Float$, $\square(Int \rightarrow Float)$, and $\square(\bigcirc Int) \oplus Int$ are stable types, but $\square Int \rightarrow Float$, $\bigcirc Int$, and $\bigcirc(\square Int)$ are not. That is, the \square modality can be used to turn any type into a stable type, thus making it possible to move functions into the future safely without risking space leaks. Using \square , we can implement the *map* function for \bigcirc :

$$\begin{aligned} \text{mapLater} &:: \square(a \rightarrow b) \rightarrow \bigcirc a \rightarrow \bigcirc b \\ \text{mapLater } f \ x &= \text{delay } (\text{unbox } f \ (\text{adv } x)) \end{aligned}$$

where *unbox* is simply a function of type $\square a \rightarrow a$.

A value of type \square can only be constructed using the introduction form *box*. The typing rule for *box*, shown in Figure 1, restricts the typing context to Γ^\square , which is obtained from Γ by removing all variables of non-stable types and all \surd_θ tokens. Hence, the type system ensures that boxed values may only refer to variables of a stable type.

2.3 Recursive definitions

Similarly to Rattus and other synchronous FRP languages [3, 26], signals can be defined in Async Rattus by the following definition:

$$\mathbf{data} \text{ Sig } a = a :: (\bigcirc(\text{Sig } a))$$

That is, a signal of type *Sig a* consists of a current value of type a and a future update to the signal of type $\bigcirc(\text{Sig } a)$. We can define a *map* function for signals, but similarly to the *mapLater* function on the \bigcirc modality, the function argument has to be boxed:

$$\begin{aligned} \text{map} &:: \square(a \rightarrow b) \rightarrow \text{Sig } a \rightarrow \text{Sig } b \\ \text{map } f \ (x :: xs) &= \text{unbox } f \ x :: \text{delay } (\text{map } f \ (\text{adv } xs)) \end{aligned}$$

In order to ensure productivity of recursive function definitions, Async Rattus requires that recursive function calls, such as *map f (adv xs)* above, have to be guarded by a *delay*. More precisely, such a recursive occurrence may only occur in a context Γ that contains a \surd_θ .

While the definition of the signal type looks superficially the same as in synchronous FRP languages, its semantics is quite different because \bigcirc has a different semantics: Updates to a signal do not come at the rate given by a global clock, but rather by some local clock, which may in turn change dynamically. For example, we can implement the constant signal function as follows:

$$\begin{aligned} \text{const} &:: a \rightarrow \text{Sig } a \\ \text{const } x &= x :: \text{never} \end{aligned}$$

where $\text{never} :: \bigcirc b$ is simply a delayed computation with a clock that will never tick. The const signal function might seem pointless, but we can combine it with a combinator that switches from one signal to another signal:

$$\begin{aligned} \text{switch} &:: \text{Sig } a \rightarrow \bigcirc(\text{Sig } a) \rightarrow \text{Sig } a \\ \text{switch } (x :: xs) \, d = x &:: \text{delay } (\text{case select } xs \, d \text{ of} \\ &\quad \text{Fst } \, xs' \, d' \rightarrow \text{switch } xs' \, d' \\ &\quad \text{Snd } \, _ \, d' \rightarrow d' \\ &\quad \text{Both } xs' \, d' \rightarrow d') \end{aligned}$$

A signal $\text{switch } s \, e$ first behaves like s , but as soon as the clock of e ticks the signal behaves like the signal produced by e . For example, given a value $x :: a$ and a delayed value $y :: \bigcirc a$, we can produce a signal that first has the value x and then, as soon as y arrives, has the value that y produces:

$$\begin{aligned} \text{step} &:: a \rightarrow \bigcirc a \rightarrow \text{Sig } a \\ \text{step } x \, y &= \text{switch } (\text{const } x) \, (\text{delay } (\text{const } (\text{adv } y))) \end{aligned}$$

2.4 Operational semantics

One of the main features of `Async Rattus` is that it avoids space leaks. To this end, its type system prevents us from moving arbitrary computations into the future. In addition, the operational semantics is carefully designed so that computations are executed as soon as the data they depend on is available. In short, this means that `Async Rattus` uses an eager evaluation semantics except for `delay` and `box`. That is, arguments are evaluated to values before they are passed on to functions, but special rules apply to `delay` and `box`. Furthermore, `Async Rattus` requires strict data types and any use of lazy data types will produce a warning.

Following the temporal interpretation of the \bigcirc modality, its introduction form delay_θ does not eagerly evaluate its argument since we have to wait until input data arrives, namely when the clock θ ticks. For example, in the following function definition, we cannot evaluate $\text{adv } x+1$ until the integer value of $x :: \bigcirc \text{Int}$ arrives, which is one time step from now on the clock $\text{cl}(x)$:

$$\begin{aligned} \text{delayInc} &:: \bigcirc \text{Int} \rightarrow \bigcirc \text{Int} \\ \text{delayInc } x &= \text{delay } (\text{adv } x + 1) \end{aligned}$$

However, evaluation is only delayed until the clock $\text{cl}(x)$ ticks, and this delay is reversed by `adv`. For example, $\text{adv } (\text{delay } (1 + 1))$ evaluates immediately to 2.

The modal FRP calculi of Krishnaswami [26] and Bahr et al. [3, 4], Bahr and Møgelberg [5] have a similar operational semantics to achieve same memory property that `Async Rattus` has. However, similarly to `Rattus`, `Async Rattus` uses a slightly more eager evaluation strategy for `delay`: Recall that $\text{delay}_\theta \, t$ delays the computation t by one time step and that `adv` reverses such a delay. The operational semantics reflects this intuition by first evaluating every term

t that occurs as $\text{delay}_{\text{cl}(t)} (\dots \text{adv } t \dots)$ before evaluating delay . In other words, $\text{delay}_{\text{cl}(t)} (\dots \text{adv } t \dots)$ is equivalent to

$$\text{let } x = t \text{ in } \text{delay}_{\text{cl}(x)} (\dots \text{adv } x \dots)$$

Similarly, $\text{delay}_{\text{cl}(s) \sqcup \text{cl}(t)} (\dots \text{select } s \ t \dots)$ is equivalent to

$$\text{let } x = s; y = t \text{ in } \text{delay}_{\text{cl}(x) \sqcup \text{cl}(y)} (\dots \text{select } x \ y \dots)$$

This generalisation of the operational semantics of delay allows us to lift some of the restrictions present in the Async RaTT calculus [5] on which Async Rattus is based: Async Rattus allows more than one \surd_{θ} in the typing context, i.e. delay can be nested; it does not prohibit lambda abstractions in the presence of a \surd_{θ} ; and both adv and select can be used with arbitrary terms, not just variables.

3 Reactive Programming in Async Rattus

In this section, we demonstrate the expressiveness of Async Rattus with a number of examples. The full source code of abridged examples along with further example programs can be found in the Async Rattus package [18].

3.1 A simple FRP application

To support FRP using the *Sig* type, we implement a library of standard FRP combinators [5] in Async Rattus. Figure 2 lists a small subset of this library.

In addition to the core language constructs of Async Rattus presented in section 2, the language also features primitives for interacting with its environment via *input channels* and *output channels*. The simplest input channel is a timer that ticks at a fixed interval (given in milliseconds):

$$\text{timer} :: \text{Int} \rightarrow \square(\circ())$$

Input channels have the type $\square(\circ a)$, but they can always be turned into signals:

$$\begin{aligned} \text{mkSig} &:: \square(\circ a) \rightarrow \circ(\text{Sig } a) \\ \text{mkSig } b &= \text{delay } (\text{adv } (\text{unbox } b) :: \text{mkSig } b) \\ \text{timerSig} &:: \text{Int} \rightarrow \text{Sig } () \\ \text{timerSig } n &= () :: \text{mkSig } (\text{timer } n) \end{aligned}$$

That is, the signal $\text{timerSig } n$ produces a new value every n microseconds. As an example, this timer input channel can be used for implementing the *derivative* and *integral* combinators in Figure 2 (as in Bahr and Møgelberg [5]).

More general input channels can be constructed using

$$\text{getInput} :: \text{IO } (\square(\circ a), (a \rightarrow \text{IO } ()))$$

which produces an input channel of type $\square(\circ a)$ that we can feed from Haskell by using the callback function of type $a \rightarrow \text{IO } ()$. Whenever the callback function

```

current  :: Sig a → a
future   :: Sig a → ○(Sig a)
map      :: □(a → b) → Sig a → Sig b
mapD    :: □(a → b) → ○(Sig a) → ○(Sig b)
const    :: a → Sig a
scan     :: (Stable b) ⇒ □(b → a → b) → b → Sig a → Sig b
zipWith  :: (Stable a, Stable b) ⇒ □(a → b → c) → Sig a → Sig b → Sig c
interleave :: □(a → a → a) → ○(Sig a) → ○(Sig a) → ○(Sig a)
switch   :: Sig a → ○(Sig a) → Sig a
switchS  :: Sig a → ○(a → Sig a) → Sig a
derivative :: Sig Float → Sig Float
integral  :: Float → Sig Float → Sig Float

```

Fig. 2. Simple FRP library.

is called with an argument v , the input channel will produce a new value v . Library authors can use `getInput` to provide an Async Rattus interface to external resources. For example, we can implement an input channel for the console with the following Haskell code:

```

consoleInput :: IO (□(○Text))
consoleInput = do (inp, cb) ← getInput
                 let loop = do line ← getLine; cb line; loop
                 forkIO loop
                 return inp

```

For output channels, Async Rattus provides the function

```

setOutput :: Sig a → (a → IO ()) → IO ()

```

which, if given a signal s and a callback function f , calls $f v$ whenever s produces a new value v . To support a variety of programming styles beyond the *Sig* type, the type of `setOutput` is in fact more general:

```

setOutput :: Producer p a ⇒ p → (a → IO ()) → IO ()

```

Instances of *Producer* $p a$ are types p that produce values of type a over time. In particular, we have instances *Producer* $(○(\text{Sig } a)) a$ and *Producer* $(\text{Sig } a) a$.

Similarly to `getInput`, library authors can use `setOutput` to provide an Async Rattus interface for external resources. For example, we may wish to provide a way to output a signal of integers by printing each new value to the console:

```

intOutput :: Producer p Int ⇒ p → IO ()
intOutput sig = setOutput sig print

```

As a simple example, we implement a console application that waits for the user to enter a line, and then outputs the length of the user's input:³

³ (\$) is the infix notation for the function $fmap :: Functor f ⇒ (a → b) → f a → f b$

```

main = do inp ← mkSignal ($) consoleInput      -- inp :: ○(Sig Text)
        let outSig = mapD (box length) inp      -- outSig :: ○(Sig Int)
            intOutput outSig
            startEventLoop

```

In the last line we call `startEventLoop :: IO ()` which starts the event loop that executes output actions registered by `setOutput`. We will look at a more comprehensive example in section 3.3.

3.2 Filtering functions

As Bahr and Møgelberg [5] have observed, the `Sig` type does not support a filter function $filter :: \Box(a \rightarrow Bool) \rightarrow Sig\ a \rightarrow Sig\ a$. The problem is that a signal of type `Sig a` must produce a value of type `a` at every tick of its current clock. In order to check the predicate $p :: \Box(a \rightarrow Bool)$ we must wait until the input signal ticks and produces a new value $v :: a$. Hence, we must produce a value for the output signal for that tick as well, regardless of whether $p\ v$ is true or not. Instead, we can implement a variant of `filter` with the following type:⁴

```

filter' :: \Box(a → Bool) → Sig a → Sig (Maybe' a)
filter' p = map (box (λx → if unbox p x then Just' x
                       else Nothing'))

```

This is somewhat unsatisfactory but workable. We can provide an implementation of standard FRP combinators (like those in Figure 2) that work with signals of type `Sig (Maybe' a)` instead of `Sig a`. However, this introduces inefficiencies since programs that work with signals of type `Sig (Maybe' a)` have to explicitly check for the `Nothing'` case for each tick.

A possible solution is to replace the modal operator \bigcirc with the derived operator F that may take several ticks to produce a result:

```

data F a = Now a | Wait (○(F a))
data Sig_F a = a ::_F (○(F (Sig_F a)))

```

That is, a value of type `F a` is the promise of a value of type `a` in 0 or more (possibly infinitely many) ticks. Then the definition of `Sig_F` replaces \bigcirc with the composition of \bigcirc and F . That is, a signal has a current value and the promise that it will update in one or more ticks. With this type, we can implement a function $filter :: \Box(a \rightarrow Bool) \rightarrow Sig_F\ a \rightarrow F\ (Sig_F\ a)$ as well as corresponding versions of the functions in Figure 2.

Sadly, `Sig_F a` still suffers from the same inefficiency as the `Sig (Maybe' a)` type. To implement a more efficient filter function, we instead make use of the `getInput` and `setOutput` functions. By composing the two, we can turn a signal of type `Sig (Maybe' a)` into a signal of type `Sig a`:

⁴ `Maybe'` is a strict variant of the standard `Maybe` type.

```

filterMap  ::  $\Box(a \rightarrow \text{Maybe}' b) \rightarrow \text{Sig } a \rightarrow \text{IO } (\Box(\bigcirc(\text{Sig } b)))$ 
filterMapD ::  $\Box(a \rightarrow \text{Maybe}' b) \rightarrow \bigcirc(\text{Sig } a) \rightarrow \text{IO } (\Box(\bigcirc(\text{Sig } b)))$ 
filter     ::  $\Box(a \rightarrow \text{Bool}) \rightarrow \text{Sig } a \rightarrow \text{IO } (\Box(\bigcirc(\text{Sig } a)))$ 
filterD    ::  $\Box(a \rightarrow \text{Bool}) \rightarrow \bigcirc(\text{Sig } a) \rightarrow \text{IO } (\Box(\bigcirc(\text{Sig } a)))$ 
trigger    ::  $(\text{Stable } a, \text{Stable } b) \Rightarrow \Box(a \rightarrow b \rightarrow c) \rightarrow \text{Sig } a \rightarrow \text{Sig } b \rightarrow \text{IO } (\Box(\text{Sig } c))$ 
triggerD   ::  $\text{Stable } b \Rightarrow \Box(a \rightarrow b \rightarrow c) \rightarrow \bigcirc(\text{Sig } a) \rightarrow \text{Sig } b \rightarrow \text{IO } (\Box(\bigcirc(\text{Sig } c)))$ 

```

Fig. 3. Filter functions in Async Rattus.

```

mkInputSig ::  $\text{Producer } p \ a \Rightarrow p \rightarrow \text{IO } (\Box(\bigcirc(\text{Sig } a)))$ 
mkInputSig p = do (out, cb)  $\leftarrow$  getInput
                  setOutput p cb
                  return box (mkSig out)

```

Since we have $\text{Producer } (\text{Sig } (\text{Maybe}' a)) \ a$, we can implement *filter* as follows:

```

filter ::  $\Box(a \rightarrow \text{Bool}) \rightarrow \text{Sig } a \rightarrow \text{IO } (\Box(\bigcirc(\text{Sig } a)))$ 
filter p xs = mkInputSig (filter' p xs)

```

Figure 3 lists further filter functions that can be implemented in this fashion: *filterMap* essentially composes the *filter* function with the *map* function. *filterMapD* $f \ xs \ ys$ is a signal that produces a new value `unbox f x y` whenever *xs* produces a new value, where *x* and *y* are the current values of *xs* and *ys* respectively. One can think of *trigger* as a left-biased version of *zipWith*. Finally, we also have versions of these functions that work with delayed signals. We will put this library to use in the next section.

3.3 Extended example

To demonstrate the use of the FRP library that we have developed above, we implement a simple interactive application. To this end, we extend our simple IO library, which so far consists of *consoleInput* and *intOutput*, with

```

setQuit ::  $(\text{Producer } p \ a) \Rightarrow p \rightarrow \text{IO } ()$ 
setQuit sig = setOutput sig  $(\lambda\_ \rightarrow \text{exitSuccess})$ 

```

which quits execution as soon as it receives the first value from the producer.

Figure 4, shows an interactive console application that uses our simple IO API. The application maintains an integer counter that increments each second (*nats*). At any time, we can show the current value of the counter by typing “show” in the console: The *showSig* signal triggers output on *showNat*. Moreover, we can manipulate the counter by either writing “negate” or a number “n” to the console, which multiplies the counter with -1 or adds *n* to it, respectively. Finally, we can quit the application by writing “quit”.

This example demonstrates the use of the different filter functions to construct new signals (see *quitSig*, *showSig*, *negSig*, *numSig*), the use of *interleave* and *trigger* to combine several signals (see *sig* and *showNat*, respectively), and the use of *switchS* to dynamically change the behaviour of a signal (see *nats'*).

```

everySecond :: Sig ()
everySecond = () :: mkSig (timer 1000000)
readInt :: Text → Maybe' Int
readInt text = case decimal text of Right (x, rest) | null rest → Just' x
                _ → Nothing'

nats :: Int → Sig Int
nats init = scan (box (λn _ → n + 1)) init everySecond
main = do
  console :: ○(Sig Text) ← mkSig ($) consoleInput
  quitSig :: ○(Sig Text) ← unbox ($) filterD (box (≡ "quit")) console
  showSig :: ○(Sig Text) ← unbox ($) filterD (box (≡ "show")) console
  negSig :: □(○(Sig Text)) ← filterD (box (≡ "negate")) console
  numSig :: □(○(Sig Int)) ← filterMapD (box readInt) console
  let sig :: □(○(Sig (Int → Int)))
      sig = box (interleave (box (○))
        (mapD (box (λ_ n → -n)) (unbox negSig))
        (mapD (box (λm n → m + n)) (unbox numSig)))
      let nats' :: Int → Sig Int
          nats' init = switchS (nats init)
              (delay (λn → nats' (current (adv (unbox sig)) n)))
  showNat :: □(○(Sig Int)) ← triggerD (box (λ_ n → n)) showSig (nats' 0)
  setQuit quitSig
  intOutput showNat
  startEventLoop

```

Fig. 4. Example reactive program.

4 Embedding Async Rattus in Haskell

The embedding of Async Rattus in Haskell consists of two main components: (1) the definition of the language's syntax in the form of standard Haskell type and function definitions, and (2) a plugin for GHC which implements the typing rules for the modal type operators and performs the necessary program transformations to obtain the desired operational semantics for Async Rattus. Since the implementation of the \square modality and the *Stable* type constraint is similar to Rattus [2], we focus our attention on the \circ modality as it requires a novel and significantly different approach due to the presence of clocks.

Syntax. Figure 5 shows the implementation of the syntax of \circ . A value *Delay* θ f of type $\circ a$ consists of a clock θ and a delayed computation f . In turn, a clock θ is a finite set of input channel identifiers and we say that θ ticks whenever any of the input channels in θ produces a new value. As soon as θ ticks, by virtue of an input channel $c \in \theta$ producing a new value v , we can run the delayed computation f by passing both c and v to f as an argument of type *InputValue*. The introduction and elimination forms *delay*, *adv*, and *select* are simply implemented as \perp . These dummy implementations are replaced by their

```

data InputValue where                                -- Not exported
    InputValue :: ChanId → a → InputValue
type Clock = Set ChanId                               -- Not exported
data  $\bigcirc a$  = Delay Clock (InputValue → a)         -- Constructor not exported
delay :: a →  $\bigcirc a$       adv ::  $\bigcirc a$  → a      select ::  $\bigcirc a$  →  $\bigcirc b$  → Select a b
delay _ =  $\perp$            adv _ =  $\perp$            select _ _ =  $\perp$ 

```

Fig. 5. Implementation of the syntax of \bigcirc .

correct implementations in a program transformation performed by the compiler plugin. The correct implementations are inserted later since they depend on compile time clocks which are inferred by the plugin and are thus not part of the surface syntax.

Scope & clock check. GHC provides a rich API that allows us to insert custom logic in several phases of its pipeline, which is sketched in Figure 6. During the type checking phase, GHC will use a constraint solver for the *Stable* type constraint provided by the plugin. Afterwards, the plugin checks the stricter scoping rules of *Async Rattus* and infers clock annotations. Checking the scoping rules for *Async Rattus* is similar to *Rattus* [2]: Variables may no longer be in scope because **delay** introduced a tick, because **adv** or **select** dropped the context to the right of a tick, or because of the restricted context Γ^\square , e.g. in the rule for **box**. The clock inference algorithm introduces an existentially quantified clock variable θ for each occurrence of **delay**, which is then instantiated to $\text{cl}(t)$ or $\text{cl}(s) \sqcup \text{cl}(t)$ as soon as it encounters an occurrence of **adv** t or **select** $s t$, respectively.

Single tick, strictness & clock transformation. GHC desugars the typed Haskell AST into the Core intermediate language, on which it then performs various simplification and optimisation steps. The *Async Rattus* plugin adds three additional transformations. Figure 7 lists the rewrite rules that are applied during these three transformations. In these rewrite rules, we use K to denote a term with a single hole that does not occur in the scope of **delay**, **adv**, **select**, **box**, or lambda abstraction, and we write $K[t]$ to denote the term obtained from K by replacing its hole with the term t .

The *single step transformation* rules preserve the typing of the program and once the rules have been exhaustively applied, the resulting program is typable with the more restrictive typing rules of the *Async RaTT* calculus [5], which only allows at most one tick in the context, requires that **adv** and **select** be applied to variables, and disallows lambda abstractions in the scope of a tick.

The *strictness transformation* replaces lambda abstractions and let bindings so that *Async Rattus* programs have a call-by-value semantics.

The clock inference performed during an earlier compilation phase established that we can find suitable clock annotation for each occurrence of **delay**. The *clock transformation* inserts these clock annotation and thereby also replaces the dummy functions **delay**, **adv**, and **select** from Figure 5 with their actual

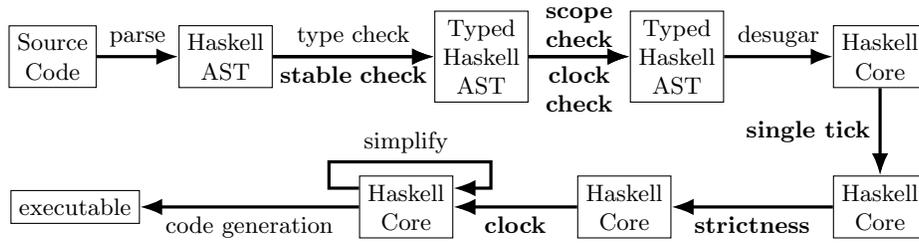


Fig. 6. Simplified pipeline of GHC extended with Async Rattus plugin (in bold).

Single tick transformation:

$$\begin{aligned}
 \text{delay}(K[\text{adv } t]) &\longrightarrow \text{let } x = t \text{ in } \text{delay}(K[\text{adv } x]) && \text{if } t \text{ is not a variable} \\
 \text{delay}(K[\text{select } s t]) &\longrightarrow \text{let } x = s \text{ in } \text{delay}(K[\text{select } x t]) && \text{if } s \text{ is not a variable} \\
 \text{delay}(K[\text{select } s t]) &\longrightarrow \text{let } x = t \text{ in } \text{delay}(K[\text{select } s x]) && \text{if } t \text{ is not a variable} \\
 \lambda x. K[\text{adv } t] &\longrightarrow \text{let } y = \text{adv } t \text{ in } \lambda x. (K[y]) \\
 \lambda x. K[\text{select } s t] &\longrightarrow \text{let } y = \text{select } s t \text{ in } \lambda x. (K[y])
 \end{aligned}$$

Strictness transformation:

$$\begin{aligned}
 \lambda x. t &\longrightarrow \lambda x. \text{case } x \text{ of } _ \rightarrow t \\
 \text{let } x = s \text{ in } t &\longrightarrow \text{case } s \text{ of } x \rightarrow t
 \end{aligned}$$

Clock transformation:

$$\begin{aligned}
 \text{delay}(K[\text{adv } x]) &\longrightarrow \text{Delay } (\text{cl}(x)) (\lambda v \rightarrow K[\text{adv}' x v]) \\
 \text{delay}(K[\text{select } x y]) &\longrightarrow \text{Delay } (\text{cl}(x) \sqcup \text{cl}(y)) (\lambda v \rightarrow K[\text{select}' x y v])
 \end{aligned}$$

Fig. 7. Transformation rules.

implementations shown in Figure 8. In addition to inserting the correct clocks, this transformation also propagates the identity of the input channel that caused the tick of the clock as well as the input value that it produced. The identity of the input channel required for the implementation of `select` as it needs to check which of the two delayed computations to advance (see `select'` in Figure 8), whereas the input value is used to implement the `getInput` function.

5 Related Work

The use of modal types for FRP has seen much attention in recent years [2–4, 10, 16, 20, 23, 24, 26, 28, 29]. The first implementation of a modal FRP language we are aware of is AdjS [25], which compiles FRP programs into JavaScript. The language is based on the synchronous modal FRP calculus of Krishnaswami [26] and uses linear types to interact with GUI widgets [27]. To address the discrepancy between the synchronous programming model of AdjS and the inherently asynchronous nature of GUIs, the λ_{Widget} calculus of Graulund

$$\begin{aligned}
\text{adv}' &:: \bigcirc a \rightarrow \text{InputValue} \rightarrow a \\
\text{adv}' (\text{Delay } _ f) \text{ inp} &= f \text{ inp} \\
\text{select}' &:: \bigcirc a \rightarrow \bigcirc b \rightarrow \text{InputValue} \rightarrow \text{Select } a \ b \\
\text{select}' a @ (\text{Delay } \theta_1 f) b @ (\text{Delay } \theta_2 g) v @ (\text{InputValue } ch \ _) \\
&= \text{if } ch \in \theta_1 \text{ then if } ch \in \theta_2 \text{ then Both } (f \ v) (g \ v) \\
&\quad \text{else Fst } (f \ v) \ b \quad \quad \quad \text{else Snd } a \ (b \ v)
\end{aligned}$$

Fig. 8. Implementation of `adv` and `select`.

et al. [16] combines linear types with an asynchronous modal type constructor \diamond . Similarly to `Async Rattus`, two values $x : \diamond A$ and $y : \diamond B$ arrive at some time in the future, but not necessarily at the same time and thus λ_{Widget} provides a `select` primitive to observe the relative arrival time. However, we are not aware of an implementation of a language based on λ_{Widget} .

`Async Rattus` is based on the `Async RaTT` calculus of Bahr and Møgelberg [5], which proposes the modal operator \ominus to model asynchronous signals (in `Async Rattus` we use the simpler notation \bigcirc instead of \ominus). Like the synchronous calculus of Krishnaswami [26] (on which `AdjS` [25] is based), but unlike the asynchronous λ_{Widget} calculus of Graulund et al. [16], `Async RaTT` comes with a proof of operational guarantees: All `Async RaTT` programs are causal, productive, and don't have space leaks. `Async Rattus` generalises the typing rules of `Async RaTT` in three ways: It allows (1) more than one tick to occur in contexts (thus allowing nested occurrences of delay), (2) function definitions to occur in the scope of ticks in the context, and (3) `adv` and `select` to be applied to arbitrary terms instead of just variables. The soundness of this generalisation is based on the single tick program transformation (cf. section 4), introduced by Bahr [2], that is performed by the compiler plugin so that the resulting program will typecheck using the stricter typing rules of `Async RaTT`. The implementation of `Async Rattus` borrows much from the implementation of `Rattus` [2], which is based on a synchronous modal FRP calculus. However, the asynchronous setting required three key additions: Inference of clocks during type checking, an additional program transformation that inserts inferred clocks into the Haskell code, and finally a new runtime system that allows `Async Rattus` and Haskell to interact. The latter is enabled by the clock transformation (cf. section 4) and is provided to the user in the form of the `getInput` and `setOutput` functions.

6 Discussion and Future Work

The implementation of `Async Rattus` as an embedded language further demonstrates the power of GHC's plugin API [12, 13, 17, 34]. Not only does it allow us to customise the type checker and use program transformations to tweak the operational semantics. We can also use it to implement program elaboration mechanisms like `Async Rattus`' clock inference.

Our goal is to use `Async Rattus` to further experiment with asynchronous modal FRP. Interesting topics for further work include: evaluation of asyn-

chronous modal FRP for implementing concurrent programs and GUI applications; library design for asynchronous modal FRP in general as well as specific problem domains; and extending the language with explicit clocks and clock quantification.

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